Frustration and time reversal symmetry breaking for Fermi and Bose-Fermi systems

Krzysztof Sacha, Katarzyna Targońska, and Jakub Zakrzewski

Instytut Fizyki imienia Mariana Smoluchowskiego, Uniwersytet Jagielloński,
ulica Reymonta 4, PL-30-059 Kraków, Poland and
Mark Kac Complex Systems Research Center, Uniwersytet Jagielloński, Kraków, Poland
(Dated: April 26, 2012)

The modulation of an optical lattice potential that breaks time-reversal symmetry enables the realization of complex tunneling amplitudes in the corresponding tight-binding model. For a superfluid Fermi gas in a triangular lattice potential with complex tunnelings the pairing function acquires a complex phase, so the frustrated magnetism of fermions can be realized. Bose-Fermi mixture of bosonic molecules and unbound fermions in the lattice shows also an interesting behavior. Due to boson-fermion coupling, the fermions become slaved by the bosons and the corresponding pairing function takes the complex phase determined by bosons. In the presence of bosons the Fermi system can reveal both gap and gapless superfluidity.

I. INTRODUCTION

Cold atoms in optical lattices provide a unique medium for mimicking effects known from other areas of physics. This is primarily due to a great flexibility and a precise manipulation of the cold atomic system [1–3]. Atoms of a fermionic or bosonic character may be placed in an optical lattice potential whose geometry may be easily controlled by changing directions and/or polarizations of laser beams. Interactions between atoms may be controlled via magnetic, optical or microwave Feshbach resonances [4, 5]. The change in the depth of the optical lattice modifies primarily the tunneling between lattice sites (having also effect on the effective interaction strength) enabling e.g. the superfluid-Mott insulator quantum phase transition in the optical realization of the Bose-Hubbard model as proposed by Jaksch and Zoller [6] and subsequently demonstrated in Ref. [7].

Another spectacular way of controlling the tunneling has been proposed by Eckardt, Weiss and Holthaus [8]. Fast periodic modulations of the optical lattice allow for an effective, time-averaged tunneling to be totally suppressed keeping the depth of the lattice potential unchanged. By varying the strength of the modulation one can induce the superfluid-Mott insulator quantum phase transition [8] as verified experimentally a few years later [9, 10]. Importantly, not only the magnitude, but also the sign of the tunneling amplitudes can be altered using this approach. This concept has been utilized in a recent proposition to create frustrated magnetism with cold bosons in a triangular lattice [11] later implemented in fascinating experiments of the Hamburg group [12].

The effective tunneling caused by periodic lattice modulations is adequately explained in the framework of the Floquet theory [8] for the periodically time-dependent Hamiltonians. The properties of the corresponding quasi-energies spectra are known to depend on the global symmetries of the Hamiltonian [13–15]. Employing the similar idea we demonstrate that periodic perturbations that break the time-reversal invariance (TRI) can change not only the sign of the tunneling amplitudes, but may also induce their complex values.

In the following, we concentrate on superfluid fermions in the Bardeen-Cooper-Schriffer (BCS) regime with broken TRI. We show that typically a pairing function for s-wave interactions acquires a complex phase which may be controlled by the TRI breaking mechanism considered in the present paper (for a discussion of p-wave orbitals see [16]). We also point out that in the Bose-Fermi mixture with broken TRI the complex ground state of bosons affects the Fermi pairing function. The effect is reminiscent of the disorder induced phase control in such mixtures as discussed by one of us recently [17]. In our case the phase control is not due to a disorder, but due to a control over the tunneling mechanism and the TRI breaking.

II. BREAKING THE TIME-REVERSAL SYMMETRY

A. One-dimensional optical lattice

Let us begin, for simplicity, with a single particle in an one-dimensional (1D) optical lattice potential driven by a double harmonic perturbation. The Hamiltonian of the system reads

$$H_0 = \frac{p^2}{2m} + V(x) + K_1 x \cos(\omega t) + K_2 x \cos(2\omega t + \varphi), \quad (1)$$

where $V(x) = V(x + a)$ is an optical lattice potential with the lattice constant a, $K_{1,2}$ stand for strengths of the driving at a basic frequency ω and its second harmonic, respectively. The Hamiltonian is time-periodic, i.e. $H_0(t + 2\pi/\omega) = H_0(t)$, and the Floquet theorem [18–20] guarantees that so called the Floquet Hamiltonian:

$$\mathcal{H} = H_0 - i\hbar\partial_t \quad (2)$$

is diagonalized by periodic functions. Eigenvalues of the $\mathcal{H}$ are referred to quasi-energies of the system analogous

to quasi-momenta in a solid state physics. They are defined modulo $\hbar\omega$ and it is sufficient to consider a single Floquet zone (an analog of the Brillouin zone). Periodic eigenfunctions of the Floquet Hamiltonian can be expanded in a Fourier series, i.e. in a basis where time t can be considered as an additional *degree of freedom*. Let us define basis vectors

$$\phi_{j,m}(x,t) = \exp\left\{-ix\left[\frac{K_1}{\omega}\sin(\omega t) + \frac{K_2}{2\omega}\sin(2\omega t + \varphi)\right]\right\}$$
$$\times \exp\left(im\omega t\right)W_j(x), \qquad (3)$$

which fulfill

$$\langle\langle\phi_{j',m'}|\phi_{j,m}\rangle\rangle = \frac{\omega}{2\pi}\int\limits_0^{2\pi/\omega} dt \int dx\, \phi^*_{j',m'}\phi_{j,m}$$
$$= \delta_{j',j}\,\delta_{m',m}, \qquad (4)$$

where m denotes a Fourier component and $W_j(x) = W(x - x_j)$ is a Wannier function of the lowest energy band localized on the j-th lattice site. The first phase factor on the r.h.s. of Eq. (3) corresponds to the unitary transformation which allows us to switch from the length gauge to the velocity gauge using quantum optics language. The matrix of the Floquet Hamiltonian consists of diagonal m-blocks in the basis (3). The blocks are very weakly coupled among themselves provided the driving frequency ω is very high. Using the tight-binding approximation and taking into an account nearest neighbor tunneling only the diagonal blocks become

$$\langle\langle\phi_{j',m}|\mathcal{H}|\phi_{j,m}\rangle\rangle = -J_{\text{eff}}\,\delta_{j',j+1} - J^*_{\text{eff}}\,\delta_{j',j-1}$$
$$+(m\omega + E_0)\,\delta_{j',j}, \qquad (5)$$

where $E_0 = \langle W_j|(p^2/2m + V)|W_j\rangle$ and the effective tunneling amplitude

$$J_{\text{eff}} = J\sum_{k=-\infty}^{\infty}\mathcal{J}_{2k}(s_1)\mathcal{J}_k(s_2)e^{ik\varphi}, \qquad (6)$$

where

$$s_i = \frac{aK_i}{\omega}, \qquad (7)$$

are dimensionless strengths of the first $(i = 1)$ and the second $(i = 2)$ harmonic, the bare tunneling amplitude is $J = -\langle W_{j+1}|(p^2/2m + V)|W_j\rangle$ and $\mathcal{J}_n$ is the ordinary Bessel function. If $\hbar\omega \gg J$ the description of a single particle system may be restricted to a single diagonal block

$$H_{\text{eff}} = \langle\langle\phi_{j',0}|\mathcal{H}|\phi_{j,0}\rangle\rangle = -J_{\text{eff}}\,\delta_{j',j+1} - J^*_{\text{eff}}\,\delta_{j',j-1}, \quad (8)$$

where the constant term E_0 has been omitted.

If there is only one harmonic present in (1) or the phase $\varphi = 0$, the Floquet Hamiltonian is time-reversal invariant and $\mathcal{H}$ is represented by a real symmetric matrix in
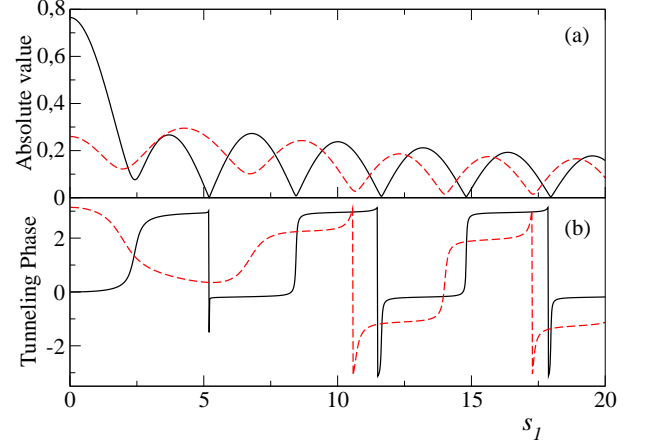

Figure 1: (Color online) The absolute value (top), A, and the complex phase φ_J (bottom) of the effective tunneling amplitude, $J_{\text{eff}}/J = A\exp(i\varphi_J)$, Eq. (6), for a double harmonic modulation of the optical lattice potential as a function of the dimensionless strength s_1 of ω component for $s_2 = 1$ [see (7)], $\varphi = 0.2$ (black solid lines) and $s_2 = 3$, $\varphi = 0.5$ (red, dashed lines).

a generic basis [21]. Then, the effective tunneling amplitude (6) is real. Single harmonic driving has been used to change the interaction from ferromagnetic (positive J_{eff}) to antiferromagnetic (negative J_{eff}) and to realize frustrated magnetic phases [11, 12]. By breaking TRI we are able to realize nearly arbitrary complex values of the tunneling amplitude $J_{\text{eff}} = |J_{\text{eff}}|e^{i\varphi_J}$. In Fig. 1 we present the absolute value $|J_{\text{eff}}|$ and the phase φ_J as a function of the parameter s_1.

The eigenstates of the Hamiltonian (8) are the Bloch waves $\psi_j = e^{ikx_j}/\sqrt{N_s}$, where N_s is the number of lattice sites, with the dispersion relation $E(k) = -2|J_{\text{eff}}|\cos(ka - \varphi_J)$. Single harmonic driving allows for $\varphi_J = 0$ or π and thus for the ground state with $k = 0$ or with k at the edge of the first Brillouin zone. The ground state of the system with broken TRI may correspond to any value of k.

We have concentrated on a 1D problem. However, a similar control of phases of the tunneling amplitudes can be also realized in higher dimensions. Indeed, the modulations applied to the lattice along orthogonal axes enable us to produce arbitrary tunnelings along the corresponding directions [11]. In the following we will focus on a 2D triangular lattice.

B. Two-dimensional triangular optical lattice

The triangular optical lattice can be realized experimentally by means of three laser beams. Single harmonic modulations of the lattice along the two orthogonal directions allows one to control the sign of the tunneling

amplitudes of particles loaded in the lattice. This setup has been used in the Hamburg experiments that demonstrated the frustrated classical magnetism [12]. With the help of the double harmonic modulations we are able to realize any phase of the tunneling amplitudes

$$J_\alpha = |J_\alpha|e^{i\varphi_\alpha}, \quad J_\beta = |J_\beta|e^{i\varphi_\beta}, \qquad (9)$$

see Fig. 2. Eigenstates of a single particle in such a lattice are Bloch waves with the dispersion relation

$$
\begin{aligned}
E(\mathbf{k}) = & -2|J_\alpha|\cos(k_x a - \varphi_\alpha) \\
& -2|J_\beta|\left\{\cos\left[\left(\sqrt{3}k_y + k_x\right)\frac{a}{2} - \varphi_\beta\right]\right. \\
& \left. +\cos\left[\left(\sqrt{3}k_y - k_x\right)\frac{a}{2} - \varphi_\beta\right]\right\}.
\end{aligned}
\qquad (10)
$$

We induce a shift of the dispersion relation along the k_y direction in the reciprocal space by changing the value of φ_β (with the other parameters fixed). The modification of φ_α alters the structure of the dispersion relation. It can reveal a doubly degenerate ground state for $\varphi_\alpha = \pi$. The presence of such a degeneracy has been observed experimentally in a Bose system [12]. For example, for $J_\alpha = J_\beta = -|J_\beta|$ the system in most experimental realizations chooses spontaneously one of the two ground states. With the double harmonic modulation breaking a TRI, the two degenerate minima for $\varphi_\alpha = \pi$ can be moved arbitrarily along the k_y direction with a change of φ_β, see Fig. 3.

In the Hamburg experiment a Bose-Einstein condensate (BEC) has been prepared in a triangular lattice. Although, in that case particle interactions are present, the ground state is still determined by the single particle dispersion relation (10). Indeed, assuming an homogeneous system (which is a good approximation of the experimental situation) the solution of the Gross-Pitaevskii equation has the chemical potential given by $\mu_B = E(\mathbf{k}) + n_B U_B$, where U_B characterizes the on-site particle interactions and n_B is an average number of bosons per a lattice site. We would like to stress, that in the presence of the interactions, the restriction to a single block of the Floquet Hamiltonian like in (8) is valid provided $\hbar\omega \gg U_B$ [8]. On the other hand, $\hbar\omega$ must be much smaller than the energy separation between bands of the periodic lattice problem for the description limited to the lowest band to be valid.

III. FERMIONS IN A TRIANGULAR LATTICE

The frustrated classical magnetism in a triangular optical lattice has been demonstrated experimentally in a Bose system [12]. Within the mean field approximation Bose-Einstein condensate wavefunction is a Bloch wave with the wavevector corresponding to the minimum of the dispersion relation (10). For the antiferromagnetic interactions the system experiences the frustration, because the tendency of the wavefunction to change the phase by
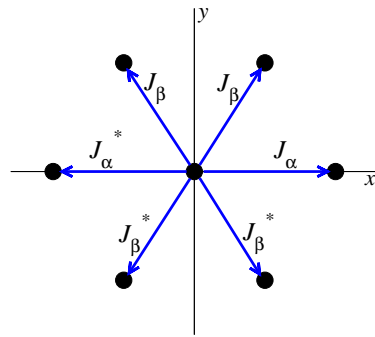

Figure 2: (Color online) Triangular Bravais lattice points (black circles) and amplitudes $J_{\alpha,\beta}$ corresponding to tunneling from a lattice point to the nearest neighbors.
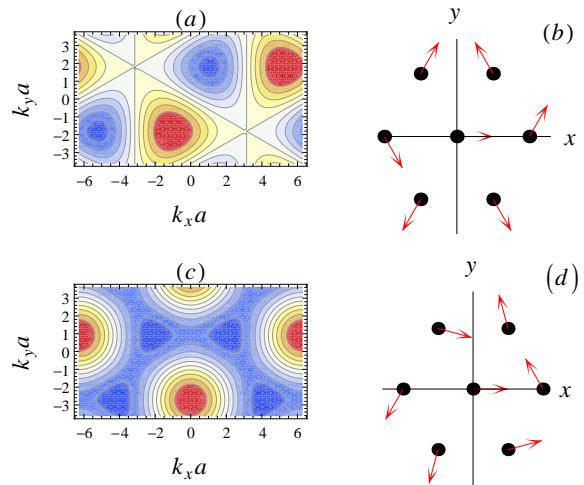

Figure 3: (Color online) Contour plots of the dispersion relation Eq. (10) for $|J_\alpha| = |J_\beta|$ and $\varphi_\alpha = \varphi_\beta = \pi/2$ (a) and $\varphi_\alpha = \pi$ and $\varphi_\beta = \pi/4$ (c); cool colors indicate regions around energy minima. In right panels directions of arrows indicate phases $e^{i\mathbf{k}\cdot\mathbf{r}_i}$ where $\mathbf{k}$ corresponds to a minimum of the dispersion relation. Specifically $\mathbf{k}a = \left(\frac{\pi}{3}, \frac{\pi}{\sqrt{3}}\right)$ for the minimum in (a) and $\mathbf{k}a = \left(+\frac{2\pi}{3}, \frac{\pi}{2\sqrt{3}}\right)$ for one of the two degenerate, non-equivalent minima in (c). Arrows in panel (b) and (d) are related to (a) and (c), respectively.

π, when we jump between neighboring sites, can not be reconciled with the triangular lattice geometry.

Consider now a mixture of fermions in the different internal states (say spin up $\uparrow$ and down $\downarrow$ states) with the attractive contact interactions in a 2D triangular optical lattice. We assume that the double harmonic modulations of the lattice allows us to adjust any phase of the complex tunneling amplitudes (9). In the tight-binding approximation the Hamiltonian of the Fermi system reads

$$\hat{H}_F = -\sum_{\langle ij\rangle} J_{ij}\left(\hat{a}_{i\uparrow}^\dagger \hat{a}_{j\uparrow} + \hat{a}_{i\downarrow}^\dagger \hat{a}_{j\downarrow}\right) - \mu\sum_i \left(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow}\right)$$
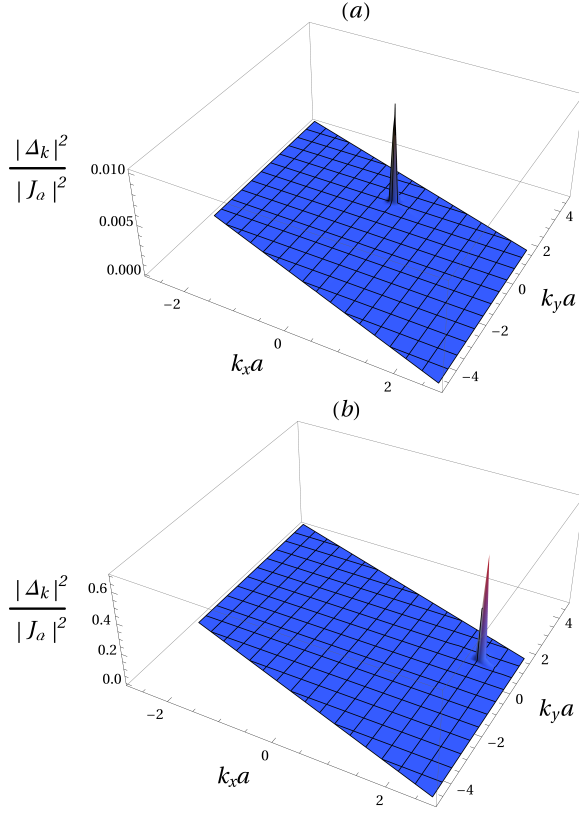
Figure 4: (Color online) Modulus squared of the Fourier transform of the BCS pairing function $|\Delta_{\mathbf{k}}|^2$, where $\Delta_{\mathbf{k}} = \sum_i \Delta_i e^{-i\mathbf{k}\cdot\mathbf{r}_i}/\sqrt{N_s}$, obtained numerically for a finite system of 60×60 lattice sites for $|J_\alpha| = |J_\beta|$, $\varphi_\alpha = \pi$, $\varphi_\beta = \pi/4$, $U/|J_\alpha| = 2$ and $\mu = 0$. Panel (a) shows $|\Delta_{\mathbf{k}}|^2$ corresponding to the ground state of the isolated Fermi system. Panel (b) presents similar results but for the Fermi system coupled to the Bose-Einstein condensate, with wavefunction $\psi_i = \sqrt{n_B} e^{i\mathbf{q}_0\cdot\mathbf{r}_i}$, where $\gamma\sqrt{n_B}/|J_\alpha| = 2.3$. Note that the peak in (a) is located at $\mathbf{k}a = (0.00, 1.78) \approx \left(0, \frac{\pi}{\sqrt{3}}\right)$ while in (b) at $\mathbf{k}a = (2.09, 0.89) \approx \mathbf{q}_0 a = \left(+\frac{2\pi}{3}, \frac{\pi}{2\sqrt{3}}\right)$.

$$-U \sum_i \hat{a}^\dagger_{i\downarrow}\hat{a}^\dagger_{i\uparrow}\hat{a}_{i\uparrow}\hat{a}_{i\downarrow}, \tag{11}$$

where the operator $\hat{a}_{i\uparrow}$ annihilates spin-up fermion at i-site, $\hat{n}_{i,\uparrow} = \hat{a}^\dagger_{i\uparrow}\hat{a}_{i\uparrow}$ and similarly for spin-down fermions. The tunneling amplitude $J_{ij} = J^*_{ji}$ and it is equal J_α or J_β, Eqs. (9), depending on a direction of the tunneling in the triangular lattice, see Fig. 2. The parameter $U > 0$ characterizes the inter-species, on-site, attractive interactions and μ stands for the chemical potential of the Fermi system.

The standard BCS approach [23] leads to the effective Hamiltonian

$$H_{F,\text{eff}} = -\sum_{\langle ij\rangle} J_{ij}\left(\hat{a}^\dagger_{i\uparrow}\hat{a}_{j\uparrow} + \hat{a}^\dagger_{i\downarrow}\hat{a}_{j\downarrow}\right) - \mu\sum_i\left(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow}\right)$$
$$+ \sum_i\left(\Delta_i\,\hat{a}^\dagger_{i\uparrow}\hat{a}^\dagger_{i\downarrow} + \Delta^*_i\,\hat{a}_{i\downarrow}\hat{a}_{i\uparrow}\right), \tag{12}$$

where the pairing function

$$\Delta_i = U\langle\hat{a}_{i,\uparrow}\hat{a}_{i,\downarrow}\rangle. \tag{13}$$

If the phases of the tunneling amplitudes (9) are zero the ground state of the system corresponds to constant pairing function $\Delta_i = \text{const}$. However, the pairing function can acquire a non-trivial phase when the tunneling amplitudes become complex. In order to find the ground state of the system let us look for the solutions of the Bogoliubov-de Gennes equations in the form

$$\begin{bmatrix} u_{\mathbf{k}}(\mathbf{r}_i) \\ v_{\mathbf{k}}(\mathbf{r}_i) \end{bmatrix} = \frac{e^{i\mathbf{k}\cdot\mathbf{r}_i}}{\sqrt{N_s}} \begin{bmatrix} U_{\mathbf{k}}\,e^{i\mathbf{k}_0\cdot\mathbf{r}_i} \\ V_{\mathbf{k}}\,e^{-i\mathbf{k}_0\cdot\mathbf{r}_i} \end{bmatrix}, \tag{14}$$

where $U_{\mathbf{k}}$ and $V_{\mathbf{k}}$ satisfy

$$\begin{bmatrix} E(\mathbf{k}+\mathbf{k}_0) - \mu & \bar{\Delta} \\ \bar{\Delta}^* & -\tilde{E}(\mathbf{k}-\mathbf{k}_0) + \mu \end{bmatrix}\begin{bmatrix} U_{\mathbf{k}} \\ V_{\mathbf{k}} \end{bmatrix} = \varepsilon_{\mathbf{k}}\begin{bmatrix} U_{\mathbf{k}} \\ V_{\mathbf{k}} \end{bmatrix}, \tag{15}$$

and $|U_{\mathbf{k}}|^2 + |V_{\mathbf{k}}|^2 = 1$. In Eqs. (15), $E(\mathbf{k})$ is the dispersion relation (10) while $\tilde{E}(\mathbf{k}) = E(\mathbf{k}; \varphi_\alpha \to -\varphi_\alpha, \varphi_\beta \to -\varphi_\beta)$. Solving (15) we obtain the following eigenvalues

$$\varepsilon_{\mathbf{k},\pm} = \frac{E(\mathbf{k}+\mathbf{k}_0) - \tilde{E}(\mathbf{k}-\mathbf{k}_0)}{2} \pm \delta\varepsilon_{\mathbf{k}}, \tag{16}$$

where

$$\delta\varepsilon_{\mathbf{k}} = \sqrt{\frac{(E(\mathbf{k}+\mathbf{k}_0) + \tilde{E}(\mathbf{k}-\mathbf{k}_0) - 2\mu)^2}{4} + |\bar{\Delta}|^2}. \tag{17}$$

The excitation spectrum of the system, i.e. the upper branch $\varepsilon_{\mathbf{k},+}$, may become negative for some $\mathbf{k}$. In such a case, the corresponding quasi-particles are present even at zero temperature. Therefore, at $T = 0$, the proper equation for $\bar{\Delta}$ reads

$$\bar{\Delta} = \frac{U}{N_s}\sum_{\mathbf{k}}\frac{\bar{\Delta}}{2\,\delta\varepsilon_{\mathbf{k}}}[1 - 2\theta(-\varepsilon_{\mathbf{k},+})], \tag{18}$$

where the Heaviside function $\theta(-\varepsilon_{\mathbf{k},+})$ ensures that quasi-particles corresponding to negative energy spectrum are also included [23]. Finally the desired pairing function becomes

$$\begin{aligned}\Delta_i &= U\sum_{\mathbf{k}} u_{\mathbf{k},+}(\mathbf{r}_i)v^*_{\mathbf{k},+}(\mathbf{r}_i)[1 - 2\theta(-\varepsilon_{\mathbf{k},+})] \\ &= e^{i2\mathbf{k}_0\cdot\mathbf{r}_i}\bar{\Delta}.\end{aligned} \tag{19}$$

When we switch from $\varphi_\alpha = \varphi_\beta = 0$ to $\varphi_\alpha = 0$ and $\varphi_\beta \neq 0$ the minimum of the dispersion relation (10) is shifted from $\mathbf{k} = 0$ to $\mathbf{k} = \mathbf{k}_0 = \left(0, \frac{2\varphi_\beta}{a\sqrt{3}}\right)$. In the ground state of the system fermions occupy energy levels starting from the new minimum up to the Fermi level. Thus, all fermions acquire quasi-momentum $\mathbf{k}_0$ and consequently the pairing function gets the quasi-momentum $2\mathbf{k}_0$, see (19).

For $|J_\alpha| = |J_\beta|$ and $\varphi_\alpha = \pi$ there are two non-equivalent, degenerate minima of $E(\mathbf{k})$. For example, for $\varphi_\beta = \pi/4$ they are located at $\mathbf{k} = \left(\pm\frac{2\pi}{3a}, \frac{\pi}{2a\sqrt{3}}\right)$, see Fig. 3c. In the ground state, fermions occupy energy levels with quasi-momenta around both of the minima. The non-zero pairing function exists for different values of $\mathbf{k}_0$. However, for $\mathbf{k}_0 = \left(0, \frac{\pi}{2a\sqrt{3}}\right)$ we obtain the lowest energy of the Hamiltonian (12). A slight change of $\mathbf{k}_0$ causes a rapid decrease of the energy gap in the excitation spectrum (16). In Fig. 4a we present Fourier transform $|\Delta_{\mathbf{k}}|^2$ of the pairing function, where $\Delta_{\mathbf{k}} = \sum_i \Delta_i e^{-i\mathbf{k}\cdot\mathbf{r}_i}/\sqrt{N_s}$, obtained numerically for a finite system. For the chemical potential $\mu = 0$, that corresponds to half filling regime for non-interacting particles, $|J_\beta|/|J_\alpha| = 1$, $U/|J_\alpha| = 2$ we obtain $|\Delta_i|/|J_\alpha| = 0.111$ at the center of the lattice which agrees with the analytical solution $\bar{\Delta}/|J_\alpha| = 0.109$ for infinite lattice. In Fig. 4a we see that even for the lattice of 60×60 sites there exists a clearly resolved peak at $\mathbf{k} \approx 2\mathbf{k}_0 = \left(0, \frac{\pi}{a\sqrt{3}}\right)$.

It was discussed [12], Bose systems can simulate the frustrated classical magnetism. We show that the similar phenomenon can be simulated in the Fermi system. Indeed, in triangular lattice with complex tunnelings, the phase of the complex pairing function is the one that is mapped onto orientation of classical spins.

IV. BOSE-FERMI MIXTURE IN A TRIANGULAR LATTICE

In this section we consider a situation when the fermions coexist with molecular dimers – pairs of spin-up and spin-down fermions. The dimers form a Bose-Einstein condensate. Such a mixture can be prepared by sweeping the system over a Feshbach resonance that creates a molecular BEC and leaves some fraction of unbound, repulsively interacting fermions. Then crossing a second Feshbach resonance one is able to change the interactions between fermions from repulsive to attractive, turn unbound fermions into BCS pairs [17]. The process does not affect the molecules at the same time. For this purpose Feshbach resonances at 202 G and 224 G for ^{40}K atoms [22] seem to be quite suitable. We also assume the presence of a weak coupling that transforms dimers into unbound fermions and vice versa. It can be realized via the photo-dissociation and photo-association. For a large molecular BEC the weak coupling does not influence significantly the condensate wavefunction and therefore we neglect dynamics of the BEC. The system under our consideration can be reduced to the following Hamiltonian

$$\hat{H} = \hat{H}_F + \hat{H}_{BF}, \tag{20}$$

with

$$\hat{H}_{BF} = \gamma \sum_i \left(\psi_i^* \, \hat{a}_{i\downarrow}\hat{a}_{i\uparrow} + \psi_i \, \hat{a}_{i\uparrow}^\dagger\hat{a}_{i\downarrow}^\dagger\right), \tag{21}$$

where the BEC wavefunction $\psi_i = \sqrt{n_B}e^{i\mathbf{q}_0\cdot\mathbf{r}_i}$ is the ground state solution for bosons in the triangular lattice, i.e. $\mathbf{q}_0$ corresponds to the minimum of the dispersion relation (10). For simplicity reasons but without loss of generality we choose the same dispersion relation for molecules and for fermions. In the system under consideration, the tunneling amplitudes for molecules in a shaken optical lattice depend on a molecular state populated in the photo-association process. The details of this process are not considered in the present paper. The coupling constant γ characterizes transfer of dimers into unbound fermions and vice versa. We consider real $\gamma \geq 0$.

In the presence of the condensate of dimers the BCS effective Hamiltonian (12) has to be supplemented with (21), that is

$$\hat{H}_{\text{eff}} = \hat{H}_{F,\text{eff}} + \hat{H}_{BF}. \tag{22}$$

In the presence of bosons, if $\mathbf{k}_0 = \mathbf{q}_0/2$, the simple analytic solution (14) of the corresponding Bogoliubov-de Gennes equations exists. This solution need not correspond to the ground state of the system. However, we will see that for sufficiently strong coupling between bosons and fermions such a solution becomes the ground state solution. Employing (14) with $\mathbf{k}_0 = \mathbf{q}_0/2$ we obtain the following equation for $\bar{\Delta}$

$$\bar{\Delta} = \frac{U}{N_s} \sum_{\mathbf{k}} \frac{\bar{\Delta} + \gamma\sqrt{n_B}}{2\delta\varepsilon_{\mathbf{k}}}[1 - 2\theta(-\varepsilon_{\mathbf{k},+})], \tag{23}$$

where, in the present case, the excitation spectrum is

$$\varepsilon_{\mathbf{k},+} = \frac{E(\mathbf{k}+\mathbf{q}_0/2) - \tilde{E}(\mathbf{k}-\mathbf{q}_0/2)}{2} + \delta\varepsilon_{\mathbf{k}}, \tag{24}$$

with

$$\delta\varepsilon_{\mathbf{k}} = \left[\frac{(E(\mathbf{k}+\mathbf{q}_0/2) + \tilde{E}(\mathbf{k}-\mathbf{q}_0/2) - 2\mu)^2}{4} + |\bar{\Delta} + \gamma\sqrt{n_B}|^2\right]^{1/2}, \tag{25}$$

and the resulting pairing function

$$\Delta_i = e^{i\mathbf{q}_0\cdot\mathbf{r}_i}\bar{\Delta}. \tag{26}$$

Let us concentrate on the triangular lattice with $|J_\beta|/|J_\alpha| = 1$, $\varphi_\alpha = \pi$ and $\varphi_\beta = \pi/4$ that corresponds to the dispersion relation plotted in Fig. 3c. The dispersion relation reveals the two non-equivalent minima, but solution of the Gross-Pitaevskii equation for bosons chooses the Bloch wave with the quasi-momentum corresponding to one of the minima. The signatures of such a spontaneous symmetry breaking are observed experimentally [12]. We assume that Bose system chooses $\mathbf{q}_0 = \left(+\frac{2\pi}{3a}, \frac{\pi}{2a\sqrt{3}}\right)$ and analyze its influence on the Fermi system.

We consider the system with $\mu = 0$. If $\gamma = 0$ the Cooper pairs with the quasi-momentum $\mathbf{q}_0$ do not exist, i.e. $\bar{\Delta} = 0$ is the only solution of (23). If the coupling between bosons and fermions is present, but $\gamma\sqrt{n_B}/|J_\alpha| < 2.112$, the system reveals gap-less superfluidity [23]. The Cooper pairs with the quasi-momentum $\mathbf{q}_0$ appear ($\bar{\Delta} \neq 0$), but there is no energy gap in the excitation spectrum. The system possesses quasi-momenta $\mathbf{k}$ for which the excitation energies $\varepsilon_{\mathbf{k},+} < 0$ and consequently the corresponding quasi-particles are present even at $T = 0$. Concerning the ground state of the system, numerical solutions of the Bogoliubov-de Gennes equations are analyzed. It is found that increasing parameter γ causes a gradual enlargement of the peak at $\mathbf{k} = \mathbf{q}_0$ in the Fourier transform of the pairing function together with a reduction of the peak at $\mathbf{k} = \left(0, \frac{\pi}{a\sqrt{3}}\right)$ (the solution in the absence of bosons considered in the previous section). For $\gamma\sqrt{n_B}/|J_\alpha| \approx 0.3$ we observe a crossover, the peak at $\mathbf{k} = \left(0, \frac{\pi}{a\sqrt{3}}\right)$ becomes hardly visible and the ground state starts to be well reproduced by the paring function (26).

For $\gamma\sqrt{n_B}/|J_\alpha| \geq 2.112$ the energy gap shows up, $\varepsilon_{\mathbf{k},+} > 0$. There is no quasi-particle at zero-temperature and the pairing function (26) is related to the ground state of the system. In Fig. 4b we show the Fourier transform of the pairing function obtained numerically for the triangular lattice of 60×60 site where the strong peak at $\mathbf{k} \approx \mathbf{q}_0$ is clearly visible. The pairing function at the center of the lattice is $|\Delta_i|/|J_\alpha| = 0.891$ and the energy gap in the excitation spectrum is $0.187|J_\alpha|$. Those numbers agree with the solutions for infinite system, i.e. $\bar{\Delta}/|J_\alpha| = 0.891$ and $\min(\varepsilon_{\mathbf{k},+}) = 0.194|J_\alpha|$.

Thus, we can describe the behaviour of the system in the following way. In a triangular optical lattice with complex tunnelings we are able to realize a BEC in the ground state with the wavevector located at an arbitrary position in the reciprocal space. If the superfluid fermions are also present in the lattice and there is the sufficiently strong coupling between fermions and bosons,

the phase of the BCS pairing function reflects the phase of the BEC wavefunction.

V. CONCLUSIONS

In summary, we have shown that the time-reversal symmetry breaking in an optical lattice potential allows us to realize complex tunneling amplitudes in the corresponding tight-binding model. We have considered a simple scheme of the symmetry breaking by means of two harmonic modulations of the lattice, but the generalization to more complicated modulations is straightforward.

We have studied a fermionic system as well as a Bose-Fermi mixture in a triangular lattice potential with complex tunnelings. In such a lattice the Bose system can simulate the frustrated classical magnetism [12]. We have shown that this behaviour is similar for fermions where the pairing function acquires a complex phase. Assuming the presence of a coupling mechanism – an exchange of unbound fermions and bosonic molecules, we have shown that the complex phase of Bose wavefunction is mapped to fermions as reflected in the Fermi pairing function.

While preparing the second, decompressed version of this manuscript we became aware of a very recent preprint [24] where the authors consider similar idea for realization of complex tunneling amplitudes in Bose systems both theoretically and experimentally.

VI. ACKNOWLEDGEMENTS

[1] D. Jaksch and P. Zoller, Ann. Phys. **315**, 52 (2005).

[2] M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen, U. Sen, Adv. Phys. **56**, 243 (2007).

[3] I. Bloch, J. Dalibard, and W. Zwerger Rev. Mod. Phys. **80**, 885 (2008).

[4] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010).

[5] T. M. Hanna, E. Tiesinga, and P. S. Julienne, New J. Phys. **12** 083031 (2010).

[6] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **81**, 3108 (1998).

[7] M. Greiner, O. Mandel, T. Esslinger, T.W. Hansch, I. Bloch, Nature **415**, 39 (2002).

[8] A. Eckardt, C. Weiss, and M. Holthaus, Phys. Rev. Lett. **95**, 260404 (2005).

[9] H. Lignier, C. Sias, D. Ciampini, Y. Singh, A. Zenesini, O. Morsch, and E. Arimondo, Phys. Rev. Lett. **99**, 220403 (2007).

[10] A. Zenesini, H. Lignier, D. Ciampini, O. Morsch, and E. Arimondo, Phys. Rev. Lett. **102**, 100403 (2009).

[11] A. Eckardt. P. Hauke, P. Soltan-Panahi, C. Becker, K. Sengstock, M. Lewenstein, Europhys. Lett. **89**, 10010 (2010).

[12] J. Struck, C. Olschlager, R. Le Targat, P. Soltan-Panahi, A. Eckardt, M. Lewenstein, P. Windpassinger, K. Sengstock, K , Science **333**, 996 (2011).

[13] K. Sacha, J. Zakrzewski and D. Delande, Phys. Rev. Lett., **83**, 2922 (1999), Erratum ibid. **84**, 200 (2000).

[14] K. Sacha, J. Zakrzewski and D. Delande, Annals of Physics (NY) **283**, 94 (2000).

[15] K. Sacha, and J. Zakrzewski, Phys. Rev. Lett. **86**, 2269 (2001).

[16] N. Bray-Ali, L. Ding, and S. Haas, Phys. Rev. B**80**, 180504(R) (2009).

[17] A. Niederberger, J. Wehr, M. Lewenstein, and K. Sacha, Europhys. Lett. **86**, 26004 (2009).

[18] G. Floquet, Ann. École Norm. Sup. **12**, 47 (1883).

[19] J. H. Shirley, Phys. Rev. **138**, B979 (1965).

[20] Y. A. Zel'dovich, Zh. Eksp. Teor. Fiz. **51**, 1492 (1966) [Sov. Phys. JETP **24**, 1006 (1967)].

[21] F. Haake, *Quantum singatures of chaos*, 2nd ed., Springer-Verlag Berlin, Heidelberg New York (2001).

[22] C. A. Regal and D. S. Jin, Phys. Rev. Lett. **90**, 230404 (2003); C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. **92**, 083201 (2004).

[23] A S Alexandrov, *Theory of Superconductivity, From Weak to Strong Coupling*, Institute of Physics Publishing Bristol and Philadelphia (2003).

[24] J. Struck, C. Ölschläger, M. Weinberg, P. Hauke, J. Simonet, A. Eckardt, M. Lewenstein, K. Sengstock, P. Windpassinger, arXiv:1203.0049.